



## Determining Hurwitz Components of $A_8$

Haval M. Mohammed Salih<sup>1</sup>

*1*Department of Mathematics, Faculty of Science, Soran University/Erbil/Iraq

Email: [havalmahmood07@gmail.com](mailto:havalmahmood07@gmail.com)

### Article info

Original: 8 May 2016  
Revised: 24 June 2016  
Accepted: 31 July 2016  
Published online: 20  
December 2016

### Key Words:

Braid orbits  
Hurwitz orbits

### Abstract

Let  $G$  be a finite group, the Hurwitz space  $H_{r,g}^{i,n}(G)$  is the space of genus  $g$  covers of the Riemann sphere with  $r$  branch points and the monodromy group  $G$ . In this paper, we give a complete list of primitive genus zero systems of  $A_8$ . We determine the braid orbits on the suitable Nielsen classes, which is equivalent to finding connected components in  $H_{r,0}^{i,n}(A_8)$ . We achieve this with the aid of the computer algebra system GAP and the MAPCLASS package.

## Introduction

Let  $R$  be a compact connected Riemann surface of genus  $g$  and that

$$\mu: R \rightarrow P^1 \tag{1}$$

is an indecomposable meromorphic function where  $P^1 = \mathbb{C} \cup \{\infty\}$  is the Riemann sphere. For every meromorphic function, there is a number  $n$  such that the fiber  $\mu^{-1}(p)$  is of  $n$  for all but finitely many points  $p \in P^1$ . The number  $n$  is called the degree of  $\mu$ . The function field  $C(R)$  is a finite algebraic extension of degree  $n$  of  $C(P^1) = C(z)$ . The points  $b$  where  $\mu^{-1}(b) < n$  are called the branch points of  $\mu$ . Let  $B \leq P^1$  be the set of branch points of  $\mu$ . It is well known that  $B$  is a finite set. So one can label the points in  $B$  by  $\{b_1, \dots, b_r\}$ . For any  $p \in P^1 \setminus B$ , the fundamental group  $\pi_1(P^1 \setminus B, p)$  acts on  $\mu^{-1}(p)$  via path lifting. Thus, one obtains a group homomorphism  $\rho: \pi_1(P^1 \setminus B, p) \rightarrow S_n$ . The image of  $\rho$  is called the monodromy group of  $\mu$  and denoted by  $Mon(R, \mu)$ . It is unique up to conjugacy in  $S_n$ . The monodromy group  $Mon(R, \mu)$  is the Galois group associated to the Galois closure of the extension  $C(R) \setminus C(P^1)$ . The homomorphism is not unique, but certain features homomorphism are unique [3]. If  $R$  is connected, then  $Mon(R, \mu)$  is a transitive subgroup of  $S_n$ . Furthermore  $\pi_1(P^1 \setminus B, p)$  is generated by all homotopy classes of loops  $\gamma_i$  winding once around the point  $b_i$ . The loops  $\gamma_i$  can be chosen so that the generators  $\gamma_i$  satisfy the only relation

$$\gamma_1 \cdot \gamma_2 \cdot \dots \cdot \gamma_r = 1. \tag{2}$$

Applying  $\rho$  to the canonical generators of  $\pi_1(P^1 \setminus B, p)$  gives the generators of a product one generating tuple in  $Mon(R, \mu)$ . Simplifying notation, by setting  $x_i = \rho(\gamma_i)$ ,  $1 \leq i \leq r$  and  $G = Mon(R, \mu)$ , then the following statements are true:

$$G = \langle x_1, x_2, \dots, x_r \rangle \tag{3}$$

$$\prod_{i=1}^r x_i = 1, x_i \in G^\#, i = 1, \dots, r. \tag{4}$$

$$\sum_{i=1}^r ind x_i = 2(n + g - 1), \tag{5}$$

Where  $ind x_i$  is the minimal number of transpositions needed to express  $x_i$  as a product.

Equation (5) is known as the Riemann-Hurwitz formula. It gives a numerical relation among  $g, n$  and  $r$ . Let  $C_i$  be the conjugacy class of  $x_i$ . Then the multi-set of non trivial conjugacy classes  $\bar{C} = \{C_1, \dots, C_r\}$  in  $G$  is called the ramification type of the cover  $\mu$ . While  $x_i$  is not uniquely determined by  $R$  and  $\mu$ , the class  $C_i$  is uniquely determined by  $R$  and  $\mu$ . This non-uniqueness will be a very interesting fact that allows us to discuss braid actions. A transitive subgroup  $G \leq S_n$  is a **genus  $g$  group** if there exist  $x_1, x_2, \dots, x_r \in G$  satisfying (3), (4) and (5) above, and we call  $(x_1, x_2, \dots, x_r)$  a **genus  $g$  system** of  $G$ . If the action of  $G$  on  $\{1, \dots, n\}$  is primitive, we call  $G$  a **primitive genus  $g$  group**  $(x_1, x_2, \dots, x_r)$  a **primitive genus  $g$  system**. Our goal is to classify primitive genus 0 systems up to diagonal conjugation and braiding for the group  $A_8$ . The Riemann Existence Theorem tells us there is a one to one correspondence between the equivalence classes of product one generating tuples  $(x_1, x_2, \dots, x_r)$  of  $G$  and the equivalence classes of  $G$ -covers of type  $\bar{C}$  such that  $x_i \in C_i$  for  $i = 1, \dots, r$ .

**Theorem 1.1 [3].** *Let  $B$  be a discrete subset of the Riemann sphere  $P^1$ . Given  $n > 1$  and a transitive permutation representation  $\rho: \pi_1(P^1 \setminus B, b_0) \rightarrow S_n$ , there is a Riemann surface  $R$  and a proper holomorphic map  $\mu: R \rightarrow P^1$  which realises  $\rho$  as its monodromy homomorphism.*

**Definition 1.2.** *Two covers  $\mu_1: X_1 \rightarrow P^1$  and  $\mu_2: X_2 \rightarrow P^1$  are equivalent if there is a homeomorphism  $h: X_1 \rightarrow X_2$  such that  $\mu_2 \circ h = \mu_1$ .*

This paper is organized as follows. Section 2, is a background results and we explain the relationship between connected components in Hurwitz spaces and braid orbits on Nielsen classes. Section 3, is devoted to label conjugacy classes of  $A_8$  according to ATLAS notation. In Section 4, we describe our methods how one can compute braid orbits for primitive genus 0 systems of  $A_8$ . Finally, we give our main results.

### Preliminaries

The equivalence classes of  $G$ -covers  $X$  of  $P^1$  with  $r$ -branched points are called a Hurwitz space and denoted by  $H_r^{in}(\bar{C})$  where  $in$  denotes an inner automorphism of  $G$ . For any  $r$ -tuple  $(x_1, x_2, \dots, x_r)$  gives a ramification type  $\bar{C}$  with  $x_i \in C_i$  for  $i = 1, \dots, r$ . Let  $\bar{C}$  be a fixed ramification type, then the subset  $H_r^{in}(\bar{C})$  of  $H_r^{in}(G)$  consists of all  $[P, \phi]$  with admissible surjective map  $\phi: \pi_1(P^1 \setminus P, p) \rightarrow G$  sends the conjugacy class  $p_i$  to the conjugacy class  $C_i$  for  $i=1, \dots, r$ . It is a union of connected components in  $H_r^{in}(G)$ . Here, we study the Hurwitz space  $H_r^{in}(G)$ . In particular we focus on the subset  $H_r^{in}(\bar{C})$  of  $H_r^{in}(G)$ .

**Definition 2.1.** For  $r \geq 2$ , the Artin braid group  $B_r$  is generated by  $r - 1$  generators  $\sigma_1 \sigma_2 \dots \sigma_{r-1}$  that satisfy the following relations:

$$\sigma_i \sigma_j = \sigma_j \sigma_i \tag{6}$$

For all  $i, j = 1, 2, \dots, r - 1$  with  $|i - j| \geq 2$ , and

$$\sigma_i \sigma_{i+1} \sigma_i = \sigma_{i+1} \sigma_i \sigma_{i+1} \tag{7}$$

For all  $i, j = 1, 2, \dots, r - 2$ . These relations are known as the braid relations.

The braid  $\sigma_i$  acts on generating tuples  $x = (x_1, x_2, \dots, x_r)$  of a finite group  $\prod_{i=1}^r x_i = 1$  as follows:

$$(x_1, \dots, x_i, x_{i+1} \dots, x_r) \sigma_i = (x_1, \dots, x_{i+1}, x_{i+1}^{-1} x_i x_{i+1} \dots, x_r), \tag{8}$$

for  $i = 1, 2, \dots, r - 1$ . The braid orbits of  $x$  is the smallest set of tuples which contains  $x$  and is closed under the operation (8).

**Definition 2.1.** The kernel of the homomorphism  $f: B_r \rightarrow S_r$ , sending  $\sigma_i$  to  $(i, i + 1)$ , is called the pure braid group. It is denoted by  $B_r^p$ .

Let  $\bar{C} = \{C_1, \dots, C_r\}$  be the ramification type of  $x = (x_1, \dots, x_r)$  where  $C_i = \{x_i^G\}$  is the conjugacy class of  $G$  containing  $x_i$ ,  $1 \leq i \leq r$ . Then each  $\sigma$  braid permutes the classes in  $\bar{C}$ , including the permutation  $f(\sigma)$  on the indices. We can assume without loss of generality that the elements in the type  $\bar{C}$  are ordered in such way

that if  $C_i = C_j$ , then for all  $i \leq k \leq j$  we have

$$C_k = C_i \tag{9}$$

We define  $\epsilon_r(G) = \{(x_1, \dots, x_r): G = \langle x_1, \dots, x_r \rangle, \prod_{i=1}^r x_i = 1, x_i \in G^\#, i = 1, 2, \dots, r\}$ .

Let  $A \leq \text{Aut}(G)$ . Then the subgroup  $A$  acts on  $\epsilon_r(G)$  via sending  $(x_1, \dots, x_r)$  to  $(a(x_1), \dots, a(x_r))$ , for  $a \in A$ , which is known as the diagonal conjugation. This action commutes with the operations (8). Thus  $A$  permutes the braid orbits. If  $A = \text{Inn}(G)$ , then it leaves each braid orbit invariant [6]. Let  $\epsilon_r^{\text{in}}(G) = \epsilon_r(G)/\text{Inn}(G)$ .

We now introduce the Nielsen classes in  $G$ , as follows. For a ramification type  $\bar{C}$ ,

$\mathcal{N}(\bar{C}) = \{(x_1, \dots, x_r): G = \langle x_1, \dots, x_r \rangle, \prod_{i=1}^r x_i = 1, \exists \sigma \in S_n \text{ such that } x_i \in C_{i\sigma} \text{ for all } i\}$ .

**Definition 2.3.** The  $r$ -th configuration space  $O_r$  of a topological space  $\mathbb{C}$ , is the space of  $r$  distinct points in  $\mathbb{C}$ .

**Lemma 2.4 [6].** The map  $\Psi_A: H_r^A(G) \rightarrow O_r, \Psi_A([P, \phi]) = P$  is covering.

The topology on the Hurwitz space  $H_r^A(G)$  completely determined by the action of the fundamental group  $\pi_1(O_r, P_0)$  where  $P_0 = \{1, \dots, r\}$  is the base point in  $O_r$  via path lifting. To be precise, we set out this action in more detail, we need a parametrization  $\Psi_A^{-1}(P_0)$ . The fiber  $\Psi_A^{-1}(P_0) = \{[P_0, \phi]_A: \phi: \pi_1(\mathbb{P}^1 \setminus P_0, \infty) \rightarrow G \text{ is admissible}\}$ . This  $\phi$  gives a product one generating tuple  $(x_1, \dots, x_r)$  of  $G$ .

**Lemma 2.5.** We obtain a bijection  $\Psi_A^{-1}(P_0) \rightarrow \epsilon_r^A(G)$  by sending  $[P_0, \phi]_A$  to the generators  $(x_1, \dots, x_r)$  where  $x_i = \phi([\gamma_i])$  for  $i = 1, \dots, r$ .

The image  $\mathcal{N}^A(\bar{C})$  of  $\mathcal{N}(\bar{C})$  is the union of braid orbits. If  $\Psi_A$  in Lemma 2.4. restricts to a connected component  $\mathcal{H}$  of  $H_r^A(G)$ , then Lemma 2.5. implies that the fiber in  $\mathcal{H}$  over  $P_0$  corresponds to the set  $\mathcal{N}^A(\bar{C})$ . For  $A = \text{Inn}(G)$ , yields a one to one correspondence between connected components of  $H_r^{\text{in}}(\bar{C})$  and the braid orbits on  $\mathcal{N}(\bar{C})$  [6].

**Definition 2.6.** Two generating tuples are braid equivalent if they lie in the same orbit under the group generated by the braid action and diagonal conjugation by  $\text{Inn}(G)$ .

**Theorem 2.7 [6].** Two generating tuples are braid equivalent if and only if their corresponding covers are equivalent.

**Definition 2.8.** If  $P$  is a partition of  $\{1, \dots, r\}$  with stabilizer  $S_P$  when  $S_P$  is a subgroup of  $S_r$ , then the full preimage of  $S_P$  is called a parabolic subgroup of  $B_r$ . It is denoted by  $B_P$ .

The following result is well known (See [3]).

**Proposition 2.9.** There is a one-to-one correspondence between connected components of  $H_r^{\text{in}}(\bar{C})$  and  $B_P$ -orbits on Nielsen classes  $\mathcal{N}(\bar{C})$ .

To answer whether or not  $\mathcal{H}_r(\bar{C})$  is connected is still an open problem, both computationally and theoretically. The computation becomes difficult when the length of Nielsen classes grows rapidly. The MAPCLASS package of James, Magaard, Shpectorov and Volklein, is designed to perform braid orbit computations for a given finite group and given type. Few results were known about it. For instance, Clebsch [1] shows that if  $G = S_n$  and let  $C = (C, \dots, C)$  be  $r$ -tuple consisting of  $r$  copies the class  $C$  of transpositions, then the corresponding Hurwitz space  $H_r^{\text{in}}(\bar{C})$  is connected. Liu and Osserman [5] generalized this result as follows. If  $G = S_n$  and  $C_i$  represented by  $x_i$  where  $x_i$  is a single cycle of length  $|x_i|$ , then  $\mathcal{H}_r(\bar{C})$  is connected. Furthermore, Fried [2] shows that if  $G = A_n, g > 0$ , and all  $C_i$  are represented by 3-cycles then  $\mathcal{H}_r(\bar{C})$ , has one component if  $g = g(X \setminus G) = 0$ , and otherwise it has two components.

Recently James A. determined all braid orbits on Nielsen classes of primitive genus zero systems for  $A_5$  and  $A_6$ .

In this paper, we collect algorithms to compute braid orbits of big length on Nielsen class. An application of these algorithm is the classification of the primitive genus zero systems for  $A_8$ . That is we find the connected components  $H_r(\bar{C})$  of  $A_8$ -curves  $X$ , such that  $g(X \setminus A_8) = 0$ . In our situation, the computation

shows that there is exactly 919 braid orbits of primitive genus 0 systems for  $A_8$ . The degree and the number of the branch points are given in Table 2.

Table 1. Number of components of  $A_8$

Degree	#Group Iso Types	# RTs	# comp's $r = 3$	# comp's $r = 4$	# comp's $r = 5$	# comp's $r = 6$	# comp's $r = 7$	# comp's total
8	1	362	649	145	87	30	8	919

**Labeling Conjugacy Classes**

Throughout this paper we will label the thirteen nontrivial conjugacy class of  $A_8$  as ATLAS notation by:

Table 2. Conjugacy Classes

Type	Coniugacy class	ind
2A	$(1, 2)(3, 4)(5, 6)(7, 8)^{A_8}$	4
2B	$(1, 2)(3, 4)^{A_8}$	2
3A	$(1, 2, 3)^{A_8}$	2
3A	$(1, 2, 3)(4, 5, 6)^{A_8}$	4
4A	$(1, 2, 3, 4)(5, 6, 7, 8)^{A_8}$	6
4B	$(1, 2, 3, 4)(5, 6)^{A_8}$	4
5A	$(1, 2, 3, 4, 5)^{A_8}$	4
6A	$(1, 2, 3)(4, 5)(6, 7)^{A_8}$	4
6B	$(1, 2, 3, 4, 5, 6)(7, 8)^{A_8}$	6
7A	$(1, 2, 3, 4, 5, 6, 7)^{A_8}$	6
7B	$(1, 2, 3, 4, 5, 6, 8)^{A_8}$	6
15A	$(1, 2, 3, 4, 5)(6, 8, 7)^{A_8}$	6
15B	$(1, 2, 3, 4, 5)(6, 7, 8)^{A_8}$	6

**Algorithm: LISTING PRIMITIVE GENUS ZERO SYSTEMS OF  $A_8$**

To obtain these tables we needed to do the following steps:

- We extract all primitive permutation group  $G$  by using the **GAP** function ALL PRIMITIVE GROUPS (Degree Operation,  $P^e$ ).
- For the group  $A_8$ , compute the conjugacy class representatives and permutation indices on 8 points.
- For given degree, genus and  $A_8$  we use the **GAP** function RestrictedPartitions to compute all possible ramification types satisfying the Riemann-Hurwitz formula.
- Compute the character table of  $A_8$  if possible and remove those types which have zero structure constant.
- For each of the remaining types of length greater than or equal to 4, we use MAPCLASS package to compute braid orbits if possible. Otherwise when the length is too big, we use Matching algorithm in [4]. For tuples of length 3 determine braid orbits via double cosets.

**Lemma 4.1.** If  $r \geq 6$ , then  $H_r^{in}(\bar{C})$  is connected.

**Lemma 4.2.** If  $r \leq 5$ , then  $H_r^{in}(\bar{C})$  is disconnected.

**Corrollary 4.3.** If  $r = 5$  and  $\bar{C} \neq (3A, 3A, 3A, 3A, 6B)$  then  $H_r^{in}(\bar{C})$  is connected.

**Corrollary 4.4.** If  $r = 4$  and  $\bar{C} \neq (3A, 3A, 5A, 6B)$  or  $(3A, 3A, 3B, 6B)$  then  $H_r^{in}(\bar{C})$  is connected.

The proof of the above results follows from the fact that the Nielsen classes  $\mathcal{N}(\bar{C})$  are the disjoint union of braid orbits but we have only one braid orbit for  $r \geq 6$  ( $r = 5$  and  $\bar{C} \neq (3A, 3A, 3A, 3A, 6B)$ ) respectively. That is  $B_r$  acts transitively on  $\mathcal{N}(\bar{C})$ . From Proposition 10.14 in [6], we obtain the Hurwitz spaces  $H_r^{in}(\bar{C})$  are connected. Otherwise the Hurwtiz spaces are disconnected.

Table 4. Connected components of  $A_8$

ramification type	number of orbit	length of largest orbit	ramification type	number of orbit	length of largest orbit
(2B,2B,2B,2B,2B,2B,2B)	1	22598400	(2B,2B,2B,2B,2B,2B,3)	1	10645740
(2B,2B,2B,2B,2B,3A,3A)	1	4918656	(2B,2B,2B,2B,3A,3A,3)	1	2208096
(2B,2B,2B,3A,3A,3A,3A)	1	974016	(2B,2B,3A,3A,3A,3A,3)	1	404160
(2B,3A,3A,3A,3A,3A,3A)	1	165120	(3A,3A,3A,3A,3A,3A,3)	1	52992
(3A,3A,3A,3A,3A,5A)	1	5200	(3A,3A,3A,3A,3A,4B)	1	19200
(3A,3A,3A,3A,3A,3B)	1	7920	(3A,3A,3A,3A,3A,6A)	1	22080
(3A,3A,3A,3A,3A,2A)	1	2400	(2B,3A,3A,3A,3A,5A)	1	16200
(2B,3A,3A,3A,3A,4B)	1	46464	(2B,3A,3A,3A,3A,3B)	1	23760
(2B,3A,3A,3A,3A,6A)	1	47424	(2B,3A,3A,3A,3A,2A)	1	4032
(2B,2B,3A,3A,3A,5A)	1	42280	(2B,2B,3A,3A,3A,4B)	1	115104
(2B,2B,3A,3A,3A,3B)	1	54396	(2B,2B,3A,3A,3A,6A)	1	103176
(2B,2B,3A,3A,3A,2A)	1	7848	(2B,2B,2B,3A,3A,5A)	1	104830
(2B,2B,2B,3A,3A,4B)	1	260592	(2B,2B,2B,3A,3A,3B)	1	125802
(2B,2B,2B,3A,3A,6A)	1	223152	(2B,2B,2B,3A,3A,2A)	1	16824
(2B,2B,2B,2B,3A,5A)	1	245800	(2B,2B,2B,2B,3A,4B)	1	585024
(2B,2B,2B,2B,3A,3B)	1	273816	(2B,2B,2B,2B,3A,6A)	1	476304
(2B,2B,2B,2B,3A,2A)	1	34110	(2B,2B,2B,2B,2B,5A)	1	557500
(2B,2B,2B,2B,2B,4B)	1	1257600	(2B,2B,2B,2B,2B,3A)	1	578800
(2B,2B,2B,2B,2B,6A)	1	1002600	(2B,2B,2B,2B,2B,2A)	1	67200

Table 4. Connected components of  $A_8$

ramification type	number of orbit	length of largest orbit	ramification type	number of orbit	length of largest orbit
(3A,3A,3A,5A,5A)	1	476	(3A,3A,3A,4B,5A)	1	1872
(3A,3A,3A,4B,4B)	1	5472	(3A,3A,3A,3B,5A)	1	876
(3A,3A,3A,3B,4B)	1	2952	(3A,3A,3A,3B,3B)	1	1080
(3A,3A,3A,6A,5A)	1	2400	(3A,3A,3A,6A,4B)	1	5904
(3A,3A,3A,6A,3B)	1	4608	(3A,3A,3A,6A,6A)	1	4608
(3A,3A,3A,3A,7B)	1	147	(3A,3A,3A,3A,7A)	1	147
(3A,3A,3A,3A,6B)	2	216	(3A,3A,3A,3A,15B)	1	165
(3A,3A,3A,3A,15A)	1	165	(3A,3A,3A,3A,4A)	1	192
(3A,3A,3A,2A,5A)	1	280	(3A,3A,3A,2A,4B)	1	480
(3A,3A,3A,2A,3B)	1	288	(3A,3A,3A,2A,6A)	1	288
(2B,3A,3A,5A,5A)	1	1556	(2B,3A,3A,4B,5A)	1	4984
(2B,3A,3A,4B,4B)	1	14112	(2B,3A,3A,3B,5A)	1	2592
(2B,3A,3A,3B,4B)	1	6468	(2B,3A,3A,3B,3B)	1	3156
(2B,3A,3A,6A,5A)	1	5320	(2B,3A,3A,6A,4B)	1	12648
(2B,3A,3A,6A,3B)	1	6084	(2B,3A,3A,6A,6A)	1	10224
(2B,3A,3A,3A,7B)	1	441	(2B,3A,3A,3A,7A)	1	441
(2B,3A,3A,3A,6B)	1	864	(2B,3A,3A,3A,15B)	1	435
(2B,3A,3A,3A,15A)	1	435	(2B,3A,3A,3A,4A)	1	384
(2B,2A,3A,3A,5A)	1	440	(2B,2A,3A,3A,4B)	1	960
(2B,2A,3A,3A,3B)	1	540	(2B,2A,3A,3A,6A)	1	720
(2B,2B,3A,5A,5A)	1	4370	(2B,2B,3A,5A,4B)	1	12640
(2B,2B,3A,4B,4B)	1	31488	(2B,2B,3A,3B,5A)	1	6230
(2B,2B,3A,3B,4B)	1	15312	(2B,2B,3A,3B,3B)	1	6828
(2B,2B,3A,6A,4B)	1	11790	(2B,2B,3A,6A,5A)	1	27312
(2B,2B,3A,6A,3B)	1	12942	(2B,2B,3A,6A,6A)	1	21504
(2B,2B,3A,3A,7B)	1	1176	(2B,2B,3A,3A,7A)	1	1176
(2B,2B,3A,3A,6B)	1	1980	(2B,2B,3A,3A,15B)	1	910
(2B,2B,3A,3A,15A)	1	910	(2B,2B,3A,3A,4A)	1	944
(2B,2B,3A,2A,5A)	1	930	(2B,2B,3A,2A,4B)	1	2088
(2B,2B,3A,2A,3B)	1	936	(2B,2B,3A,2A,6A)	1	1536
(2B,2B,2B,5A,5A)	1	11550	(2B,2B,2B,4B,5A)	1	29700
(2B,2B,2B,4B,4B)	1	70368	(2B,2B,2B,3B,5A)	1	14400
(2B,2B,2B,3B,4B)	1	31752	(2B,2B,2B,3B,3B)	1	13716
(2B,2B,2B,6A,5A)	1	25950	(2B,2B,2B,6A,4B)	1	58068
(2B,2B,2B,6A,3B)	1	27000	(2B,2B,2B,6A,6A)	1	45000
(2B,2B,2B,3A,7B)	1	2940	(2B,2B,2B,3A,7A)	1	2940
(2B,2B,2B,3A,6B)	1	2940	(2B,2B,2B,3A,15B)	1	4536
(2B,2B,2B,3A,15A)	1	2040	(2B,2B,2B,3A,4A)	1	1920
(2B,2B,2B,2A,5A)	1	1950	(2B,2B,2B,2A,4B)	1	3744
(2B,2B,2B,2A,3B)	1	1512	(2B,2B,2B,2A,6A)	1	3096
(2B,2B,2B,2B,7B)	1	6762	(2B,2B,2B,2B,7A)	1	6762
(2B,2B,2B,2B,6B)	1	9504	(2B,2B,2B,2B,15B)	1	4275
(2B,2B,2B,2B,15A)	1	4275	(2B,2B,2B,2B,4A)	1	3648

Table 5. Connected Components of  $A_8$

ramification type	number of orbit	length of largest orbit	ramification type	number of orbit	length of largest orbit
(3A,5A,5A,5A)	1	36	(3A,4B,5A,5A)	1	176
(3A,4B,4B,5A)	1	600	(3A,4B,4B,4B)	1	1800
(3A,3B,5A,5A)	1	96	(3A,3B,4B,5A)	1	316
(3A,3B,4B,4B)	1	774	(3A,3B,3B,5A)	1	124
(3A,3B,3B,4B)	1	408	(3A,3B,3B,3B)	1	144
(3A,6A,5A,5A)	1	260	(3A,6A,4B,5A)	1	688
(3A,6A,4B,4B)	1	1536	(3A,6A,3B,5A)	1	344
(3A,6A,3B,4B)	1	744	(3A,6A,3B,3B)	1	372
(3A,6A,6A,5A)	1	528	(3A,6A,6A,4B)	1	1248
(3A,6A,6A,3B)	1	564	(3A,6A,6A,6A)	1	936
(3A,3A,5A,7B)	1	14	(3A,3A,5A,7A)	1	14
(3A,3A,5A,6B)	2	24	(3A,3A,5A,15B)	1	22
(3A,3A,5A,15A)	1	22	(3A,3A,5A,4A)	1	24
(3A,3A,4B,7B)	1	56	(3A,3A,4B,7A)	1	56
(3A,3A,4B,6B)	1	120	(3A,3A,4B,15B)	1	64
(3A,3A,4B,15A)	1	64	(3A,3A,4B,4A)	1	48
(3A,3A,3B,7B)	1	28	(3A,3A,3B,7A)	1	28
(3A,3A,3B,6B)	2	36	(3A,3A,3B,15B)	1	22
(3A,3A,3B,15A)	1	22	(3A,3A,3B,4A)	1	28
(3A,3A,6A,7B)	1	70	(3A,3A,6A,7A)	1	70
(3A,3A,6A,6B)	1	96	(3A,3A,6A,15B)	1	50
(3A,3A,6A,15A)	1	50	(3A,3A,6A,4A)	1	56
(3A,3A,2A,7B)	1	7	(3A,3A,2A,7A)	1	7
(3A,3A,2A,6B)	1	5	(3A,3A,2A,15B)	1	5
(3A,2A,5A,5A)	1	32	(3A,2A,4B,5A)	1	52
(3A,2A,4B,4B)	1	120	(3A,2A,3B,5A)	1	40
(3A,2A,3B,4B)	1	66	(3A,2A,3B,3B)	1	24
(3A,2A,6A,5A)	1	32	(3A,2A,6A,4B)	1	84
(3A,2A,6A,3B)	1	36	(3A,2A,6A,6A)	1	72
(2B,5A,5A,5A)	1	120	(2B,4B,5A,5A)	1	520
(2B,4B,4B,5A)	1	1570	(2B,4B,4B,4B)	1	3768
(2B,3B,5A,5A)	1	300	(2B,3B,4B,5A)	1	750
(2B,3B,4B,4B)	1	1770	(2B,3B,3B,5A)	1	340
(2B,3B,3B,4B)	1	648	(2B,3B,3B,3B)	1	180
(2B,6A,5A,5A)	1	610	(2B,6A,4B,5A)	1	1480
(2B,6A,4B,4B)	1	3384	(2B,6A,3B,5A)	1	730
(2B,6A,3B,4B)	1	1590	(2B,6A,3B,3B)	1	714
(2B,6A,6A,5A)	1	1190	(2B,6A,6A,4B)	1	2616
(2B,6A,6A,3B)	1	1242	(2B,6A,6A,6A)	1	1944
(2B,3A,5A,7B)	1	49	(2B,3A,5A,7A)	1	49
(2B,3A,5A,6B)	1	96	(2B,3A,5A,15B)	1	53
(2B,3A,5A,15A)	1	53	(2B,3A,5A,4A)	1	52
(2B,3A,4B,7B)	1	154	(2B,3A,4B,7A)	1	154
(2B,3A,4B,6B)	1	270	(2B,3A,4B,15B)	1	122
(2B,3A,4B,15A)	1	122	(2B,3A,4B,4A)	1	132
(2B,3A,3B,7B)	1	77	(2B,3A,3B,7A)	1	77
(2B,3A,3B,6B)	1	138	(2B,3A,3B,15B)	1	61
(2B,3A,3B,15A)	1	61	(2B,3A,3B,4A)	1	46
(2B,3A,6A,7B)	1	161	(2B,3A,6A,7A)	1	161
(2B,3A,6A,6B)	1	240	(2B,3A,6A,15B)	1	105
(2B,3A,6A,15A)	1	105	(2B,3A,6A,4A)	1	108
(2B,3A,2A,7B)	1	14	(2B,3A,2A,7A)	1	14
(2B,3A,2A,15B)	1	10	(2B,3A,2A,15A)	1	10
(2B,2A,5A,5A)	1	50	(2B,2A,4B,5A)	1	120
(2B,2A,4B,4B)	1	216	(2B,2A,3B,5A)	1	60
(2B,2A,3B,4B)	1	60			
(2B,2A,6A,5A)	1	90	(2B,2A,6A,4B)	1	180
(2B,2A,6A,3B)	1	72	(2B,2A,6A,6A)	1	144
(2B,2B,5A,7B)	1	140	(2B,2B,5A,7A)	1	140
(2B,2B,6A,6B)	1	240	(2B,2B,6A,15B)	1	110
(2B,2B,6A,15A)	1	110	(2B,2B,6A,4A)	1	110
(2B,2B,4B,7B)	1	364	(2B,2B,4B,7A)	1	364
(2B,2B,4B,6B)	1	552	(2B,2B,4B,15B)	1	270
(2B,2B,4B,15A)	1	270	(2B,2B,4B,4A)	1	192
(2B,2B,3B,7B)	1	154	(2B,2B,3B,7A)	1	154
(2B,2B,3B,6B)	1	216	(2B,2B,3B,15B)	1	115
(2B,2B,3B,15A)	1	115	(2B,2B,3B,4A)	1	56

Table 6. Connected Components of  $A_8$

ramification type	number of orbit	length of largest orbit	ramification type	number of orbit	length of largest orbit
(2B,2B,6A,7B)	1	364	(2B,2B,6A,7A)	1	364
(2B,2B,6A,6B)	1	504	(2B,2B,6A,15B)	1	220
(2B,2B,6A,15A)	1	220	(2B,2B,6A,4A)	1	206
(2B,2B,2A,7B)	1	14	(2B,2B,2A,7A)	1	14
(2B,2B,2A,15B)	1	15	(2B,2B,2A,15A)	1	15

Table 7. Connected Components of  $A_8$

ramification type	number of orbit	length of largest orbit	ramification type	number of orbit	length of largest orbit
(5A,5A,7B)	1	1	(5A,5A,7A)	1	1
(5A,5A,6B)	4	1	(5A,5A,15B)	3	1
(5A,5A,15A)	3	1	(5A,5A,4A)	4	1
(4B,5A,7B)	6	1	(4B,5A,7A)	6	1
(4B,5A,6B)	14	1	(4B,5A,15B)	8	1
(4B,5A,15A)	8	1	(4B,5A,4A)	6	1
(4B,4B,7B)	18	1	(4B,4B,7A)	18	1
(4B,4B,6B)	32	1	(4B,4B,15B)	15	1
(4B,4B,15A)	15	1	(4B,4B,4A)	12	1
(3B,5A,7B)	3	1	(3B,5A,7A)	3	1
(3B,5A,6B)	10	1	(3B,5A,15B)	3	1
(3B,5A,15A)	3	1	(3B,5A,4A)	2	1
(3B,4B,7B)	6	1	(3B,4B,7A)	6	1
(3B,4B,6B)	10	1	(3B,4B,15B)	8	1
(3B,4B,15A)	8	1	(3B,3B,15B)	2	1
(3B,3B,15A)	2	1			1
(6A,5A,7B)	9	1	(6A,5A,7A)	9	1
(6A,5A,6B)	12	1	(6A,5A,15B)	5	1
(6A,5A,15A)	5	1	(6A,5A,4A)	8	1
(6A,4B,7B)	22	1	(6A,4B,7A)	22	1
(6A,4B,6B)	30	1	(6A,4B,15B)	14	1
(6A,4B,15A)	14	1	(6A,4B,4A)	12	1
(6A,3B,7B)	11	1	(6A,3B,7A)	11	1
(6A,3B,6B)	12	1	(6A,3B,15B)	7	1
(6A,3B,15A)	7	1	(6A,3B,4A)	4	1
(6A,6A,7B)	17	1	(6A,6A,7A)	17	1
(6A,6A,6B)	24	1	(6A,6A,15B)	9	1
(6A,6A,15A)	9	1	(6A,6A,4A)	12	1
(3A,7B,7B)	1	1	(3A,7A,7A)	1	1
(3A,6B,7B)	2	1	(3A,6B,7A)	2	1
(3A,15B,7B)	1	1	(3A,15B,7A)	1	1
(3A,15B,6B)	2	1	(3A,15B,15B)	1	1
(3A,15A,7B)	1	1	(3A,15A,7A)	1	1
(3A,15A,6B)	2	1	(3A,15A,15A)	1	1
(3A,4A,7B)	1	1	(3A,4A,7A)	1	1
(3A,4A,15B)	1	1	(3A,4A,15A)	1	1
(3A,5A,7B)	1	1	(3A,5A,7A)	1	1
(3A,5A,15B)	1	1	(3A,5A,15A)	1	1
(3A,4B,15B)	1	1	(3A,4B,15A)	1	1
(3A,6A,7B)	1	1	(3A,6A,7A)	1	1
(3A,6A,15B)	1	1	(3A,6A,15A)	1	1
(2B,7A,7B)	3	1	(2B,6B,15B)	2	1
(2B,6B,15A)	2	1	(2B,15B,7B)	2	1
(2B,15B,7A)	2	1	(2B,15B,6B)	3	1
(2B,15B,15B)	1	1	(2B,15A,7B)	2	1
(2B,15A,7A)	2	1	(2B,15A,6B)	3	1
(2B,15A,15B)	2	1	(2B,15A,15A)	1	1
(2B,4A,15B)	1	1	(2B,4A,15A)	1	1

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